Sequential Logic

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Outline

- Memory Elements
- Latches: *S*-*R* Latch, *D* Latch
- Flip-flops: *S*-*R* flip-flop, *D* flip-flop, *J*-*K* flip-flops, *T* flip-flops
- Asynchronous Inputs
- Synchronous Sequential Circuit Analysis

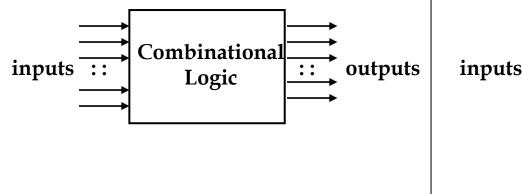
Note: These slides are taken from Aaron Tan's slide

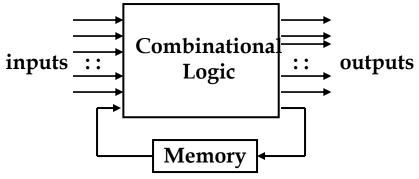
Introduction (1/2)

- Two classes of logic circuits
 - Combinational
 - Sequential
- Combinational Circuit
 - Each output depends entirely on the immediate (present) inputs.



• Each output depends on both present inputs and state.



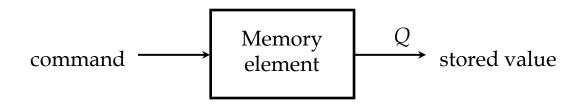


Introduction (2/2)

- Two types of sequential circuits:
 - Synchronous: outputs change only at specific time
 - Asynchronous: outputs change at any time
- Multivibrator: a class of sequential circuits
 - Bistable (2 stable states)
 - Monostable or one-shot (1 stable state)
 - Astable (no stable state)
- Bistable logic devices
 - Latches and flip-flops.
 - They differ in the methods used for changing their state.

Memory Elements (1/3)

• Memory element: a device which can remember value indefinitely, or change value on command from its inputs.



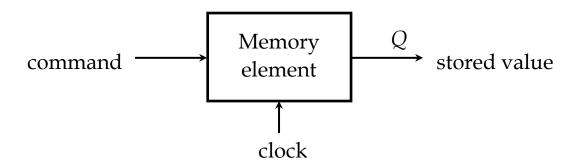
Characteristic table:

Command (at time <i>t</i>)	Q(t)	Q(t+1)
Set	Х	1
Reset	Х	0
Memorise /	0	0
No Change	1	1

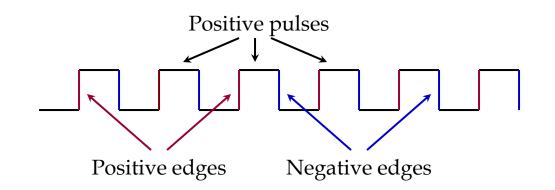
Q(*t*) or *Q*: current state *Q*(*t*+1) or *Q*⁺: next state



• Memory element with clock.



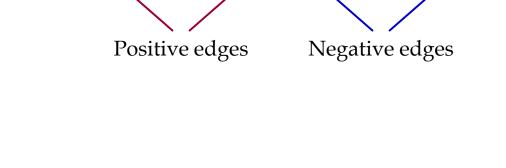
• Clock is usually a square wave.



Memory Elements (3/3)

- Two types of triggering/activation
 - Pulse-triggered
 - Edge-triggered
- Pulse-triggered
 - Latches
 - ON = 1, OFF = 0
- Edge-triggered
 - Flip-flops
 - Positive edge-triggered (ON = from 0 to 1; OFF = other time)
 - Negative edge-triggered (ON = from 1 to 0; OFF = other time)





Positive pulses

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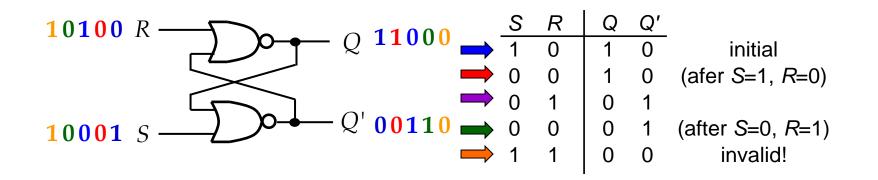
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S-R Latch (1/3)

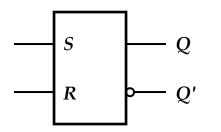
- Two inputs: *S* and *R*.
- Two complementary outputs: *Q* and *Q*'.
 - When Q = HIGH, we say latch is in SET state.
 - When Q = LOW, we say latch is in RESET state.
- For active-high input *S*-*R* latch (also known as NOR gate latch)
 - R = HIGH and S = LOW \rightarrow Q becomes LOW (RESET state)
 - *S* = HIGH and *R* = LOW \rightarrow *Q* becomes HIGH (SET state)
 - Both *R* and *S* are LOW \rightarrow No change in output *Q*
 - Both *R* and *S* are HIGH →Outputs *Q* and *Q*' are both LOW (invalid!)
- Drawback: invalid condition exists and must be avoided.



• Active-high input *S*-*R* latch:

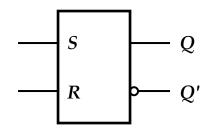


• Block diagram:





• Characteristic table for active-high input *S-R* latch:



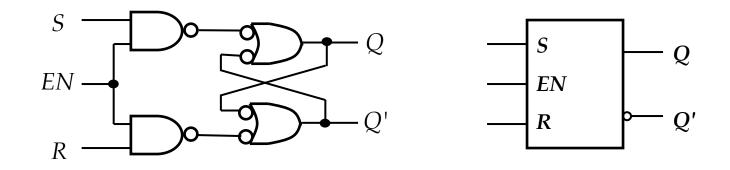
S	R	Q	Q'	
0	0	NC	NC	No change. Latch remained in present state.
1	0	1	0	Latch SET.
0	1	0	1	Latch RESET.
1	1	0	0	Invalid condition.

S	R	Q(t+1)		
0	0	Q(t)	No change	C
0	1	0	Reset	×
1	0	1	Set	
1	1	indeterminate		

$$Q(t+1) = ?$$

Gated S-R Latch

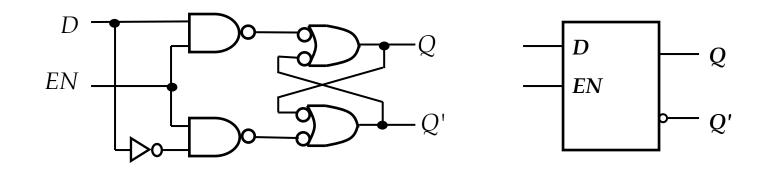
• *S*-*R* latch + *enable input* (*EN*) and 2 NAND gates → a gated *S*-*R* latch.



• Outputs change (if necessary) only when *EN* is high.

Gated D Latch (1/2)

- Make input *R* equal to $S' \rightarrow \text{gated } D$ latch.
- *D* latch eliminates the undesirable condition of invalid state in the *S*-*R* latch.



Gated D Latch (2/2)

- When *EN* is high,
 - − D = HIGH → latch is SET
 - − $D = LOW \rightarrow latch is RESET$
- Hence when EN is high, *Q* "follows" the *D* (data) input.
- Characteristic table:

EN	D	Q(t+1)	
1	0	0	Reset
1	1	1	Set
0	Χ	Q(t)	No change

When EN=1, Q(t+1) = ?

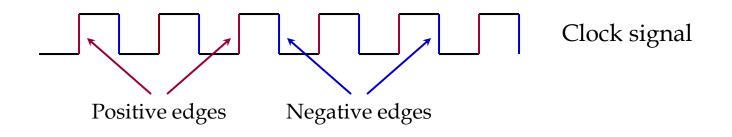
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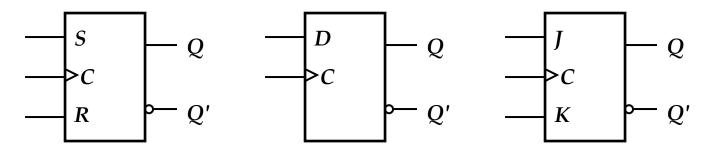
Flip-Flops (1/2)

- Flip-flops are synchronous bistable devices.
- Output changes state at a specified point on a triggering input called the clock.
- Change state either at the positive (rising) edge, or at the negative (falling) edge of the clock signal.

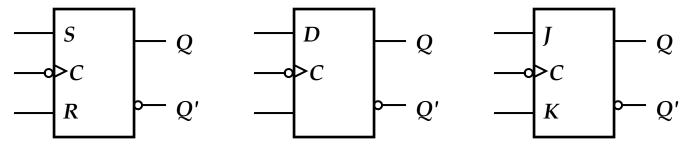


Flip-Flops (2/2)

- *S*-*R* flip-flop, *D* flip-flop, and *J*-*K* flip-flop.
- Note the ">" symbol at the clock input.



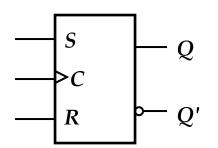
Positive edge-triggered flip-flops



Negative edge-triggered flip-flops

S-R Flip-Flop

- *S-R* flip-flop: On the triggering edge of the clock pulse,
 - R = HIGH and S = LOW \rightarrow Q becomes LOW (RESET state)
 - *S* = HIGH and *R* = LOW \rightarrow *Q* becomes HIGH (SET state)
 - Both *R* and *S* are LOW \rightarrow No change in output *Q*
 - Both *R* and *S* are HIGH \rightarrow Invalid!
- Characteristic table of positive edge-triggered *S*-*R* flip-flop:



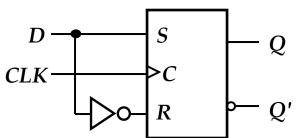
S	R	CLK	Q(t+1)	Comments
0	0	Х	<i>Q(t)</i>	No change
0	1	1	0	Reset
1	0	1	1	Set
1	1	1	?	Invalid

X = irrelevant ("don't care")

 \uparrow = clock transition LOW to HIGH

D Flip-Flop (1/2)

- *D* flip-flop: Single input *D* (data). On the triggering edge of the clock pulse,
 - $D = HIGH \rightarrow Q$ becomes HIGH (SET state)
 - $D = LOW \rightarrow Q$ becomes LOW (RESET state)
- Hence, *Q* "follows" *D* at the clock edge.
- Convert *S*-*R* flip-flop into a *D* flip-flop: add an inverter.

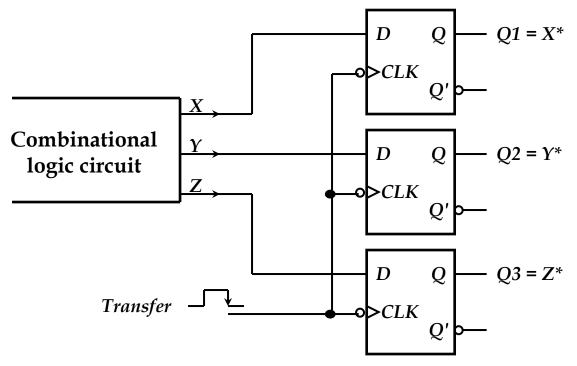


D	CLK	Q(t+1)	Comments
1	1	1	Set
0	1	0	Reset

A positive edge-triggered D flipflop formed with an S-R flipflop. \uparrow = clock transition LOW to HIGH



- Application: Parallel data transfer.
 - To transfer logic-circuit outputs *X*, *Y*, *Z* to flip-flops *Q*1, *Q*2 and *Q*3 for storage.



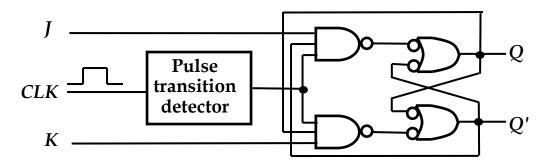
* After occurrence of negative-going transition

J-K Flip-Flop (1/2)

- *J-K* flip-flop: *Q* and *Q*' are fed back to the pulse-steering NAND gates.
- No invalid state.
- Include a toggle state
 - $J = HIGH and K = LOW \rightarrow Q$ becomes HIGH (SET state)
 - K = HIGH and J = LOW $\rightarrow Q$ becomes LOW (RESET state)
 - Both *J* and *K* are LOW \rightarrow No change in output *Q*
 - Both *J* and *K* are HIGH \rightarrow Toggle



• *J-K* flip-flop circuit:



• Characteristic table:

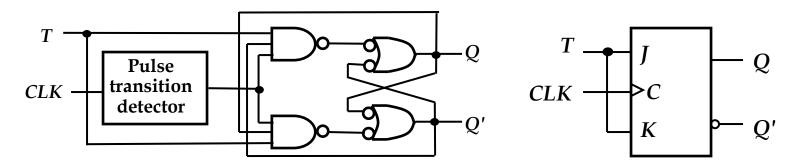
J	Κ	CLK	Q(t+1)	Comments
0	0	1	<i>Q(t)</i>	No change
0	1	1	0	Reset
1	0	1	1	Set
1	1	\uparrow	Q(t)'	Toggle

Q(*t*+1) = ?

Q	J	Κ	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



• *T* flip-flop: Single input version of the *J*-*K* flip-flop, formed by tying both inputs together.



• Characteristic table:

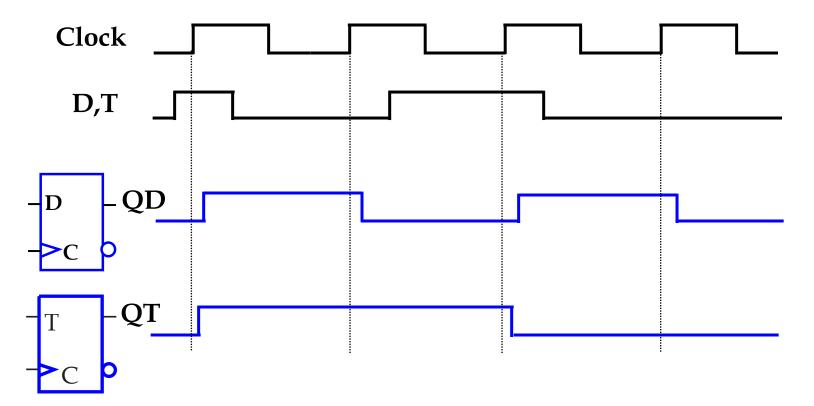
Τ	CLK	Q(t+1)	Comments
0	1	<i>Q(t)</i>	No change
1	\uparrow	Q(t)'	Toggle

Q	Τ	Q(t+1)
0	0	0
0	1	1
1	0	1
1	1	0

Q(t+1) = ?

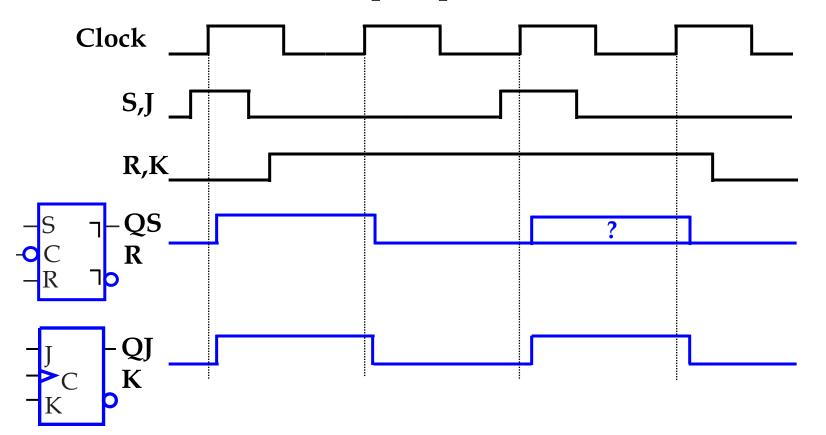
Flip-flop Behavior Example

• Use the characteristic tables to find the output waveforms for the flip-flops shown:



Flip-Flop Behavior Example (continued)

• Use the characteristic tables to find the output waveforms for the flip-flops shown:



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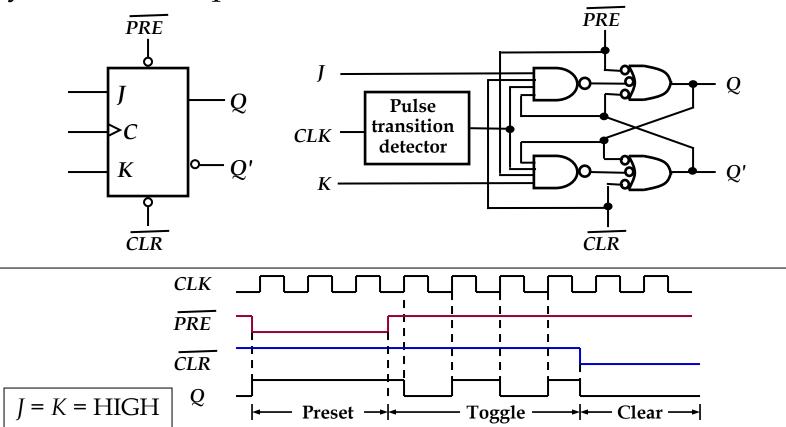
Note: These slides are taken from Aaron Tan's slide

Asynchronous Inputs (1/2)

- *S*-*R*, *D* and *J*-*K* inputs are synchronous inputs, as data on these inputs are transferred to the flip-flop's output only on the triggered edge of the clock pulse.
- Asynchronous inputs affect the state of the flip-flop independent of the clock; example: *preset* (*PRE*) and *clear* (*CLR*) [or *direct set* (*SD*) and *direct reset* (*RD*)].
- When *PRE*=HIGH, *Q* is <u>immediately</u> set to HIGH.
- When *CLR*=HIGH, *Q* is <u>immediately</u> cleared to LOW.
- Flip-flop in normal operation mode when both *PRE* and *CLR* are LOW.

Asynchronous Inputs (2/2)

• A *J*-*K* flip-flop with active-low PRESET and CLEAR asynchronous inputs.



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Synchronous Sequential Circuits

- Building blocks: logic gates and flip-flops.
- Flip-flops make up the memory while the gates form one or more combinational subcircuits.
- We have discussed *S*-*R* flip-flop, *J*-*K* flip-flop, *D* flip-flop and *T* flip-flop.

Flip-Flop Characteristic Tables

• Each type of flip-flop has its own behaviour, shown by its characteristic table.

J	K	Q(t+1)	Comments	-	S	R
0	0	<i>Q(t)</i>	No change	_	0	0
0	1	0	Reset		0	1
1	0	1	Set		1	0
1	1	Q(t)'	Toggle		1	1

	S	R	Q(t+1)	Comments
	0	0	<i>Q(t)</i>	No change
(0	1	0	Reset
1	1	0	1	Set
1	1	1	?	Unpredictable

D	Q(t+1)	_
0	0	Reset
1	1	Set

Τ	Q(t+1)	
0	<i>Q(t)</i>	No change
1	Q(t)'	Toggle

Sequential Circuits: Analysis (1/7)

- Given a sequential circuit diagram, we can analyze its behaviour by deriving its *state table* and hence its *state diagram*.
- Requires *state equations* to be derived for the flipflop inputs, as well as *output functions* for the circuit outputs other than the flip-flops (if any).
- We use *A*(*t*) and *A*(*t*+1) (or simply *A* and *A*⁺) to represent the present state and next state, respectively, of a flip-flop represented by *A*.

Sequential Circuits: Analysis (2/7)

• Example using *D* flip-flops

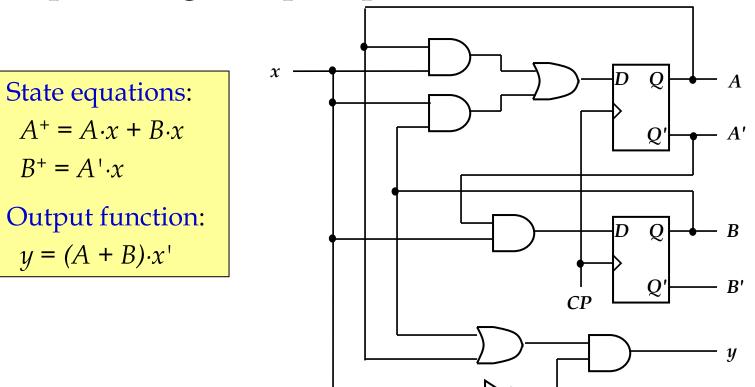


Figure 1

Sequential Circuits: Analysis (3/7)

- From the *state equations* and *output function*, we derive the *state table*, consisting of all possible binary combinations of present states and inputs.
- State table
 - Similar to truth table.
 - Inputs and present state on the left side.
 - Outputs and next state on the right side.
- *m* flip-flops and *n* inputs $\rightarrow 2^{m+n}$ rows.

Sequential Circuits: Analysis (4/7)

• *State table* for circuit of Figure 1:

State equations:			Output function:					
$A^+ = A \cdot x + B \cdot x$				$y = (A + B) \cdot x'$				
$B^+ = A' \cdot x$								
	Present			Next				
	State		Input	Sta	ate	<u>Output</u>		
	Α	В	X	A ⁺	B^{+}	У		
	0	0	0	0	0	0		
	0	0	1	0	1	0		
	0	1	0	0	0	1		
	0	1	1	1	1	0		
	1	0	0	0	0	1		
	1	0	1	1	0	0		
	1	1	0	0	0	1		
	1	1	1	1	0	0		

Sequential Circuits: Analysis (5/7)

• Alternative form of state table:

	Present State		Inpu		Next State		Output	
	Α	В	X	Ā	+		у	
Full table	0	0	0	()	0	0	
	0	0	1	()	1	0	
	0	1	0	()	0	1	
	0	1	1	1		1	0	
	1	0	0	()	0	1	
	1	0	1	1		0	0	
	1	1	0	()	0	1	
	1	1	1	-	I	0	0	_
	_	Present State		t Next State			Output	
Compact table				<i>x</i> =0 <i>x</i> =1		<i>x</i> =0	<i>x</i> =1	
Compact table		AB		A ⁺ B ⁺	A ⁺	B ⁺	У	У
		00		00	0	1	0	0
		01		00	1	1	1	0
		10		00	1	0	1	0
		11		00	1	0	1	0

Sequential Circuits: Analysis (6/7)

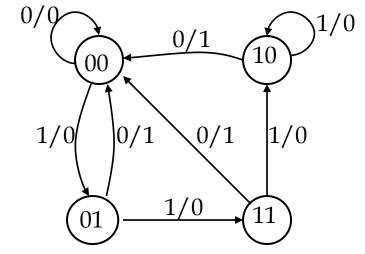
- From the *state table*, we can draw the *state diagram*.
- State diagram
 - Each state is denoted by a circle.
 - Each arrow (between two circles) denotes a transition of the sequential circuit (a row in state table).
 - A label of the form *a/b* is attached to each arrow where *a* (if there is one) denotes the inputs while *b* (if there is one) denotes the outputs of the circuit in that transition.
- Each combination of the flip-flop values represents a state. Hence, *m* flip-flops \rightarrow up to 2^{*m*} states.

Sequential Circuits: Analysis (7/7)

• **State diagram** of the circuit of Figure 1:

Present	Next	State	Output				
State	<i>x</i> =0	<i>x</i> =1	<i>x</i> =0	<i>x</i> =1			
AB	A ⁺ B ⁺	<i>A</i> ⁺ <i>B</i> ⁺	У	У			
00	00	01	0	0			
01	00	11	1	0			
10	00	10	1	0			
11	00	10	1	0			

JE!



Flip-Flop Input Function (1/3)

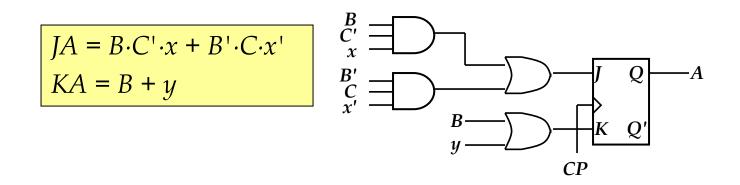
• The outputs of a sequential circuit are functions of the present states of the flip-flops and the inputs. These are described algebraically by the *circuit output functions*.

- In Figure 1: $y = (A + B) \cdot x'$

- The part of the circuit that generates inputs to the flipflops are described algebraically by the *flip-flop input functions* (or *flip-flop input equations*).
- The flip-flop input functions determine the next state generation.
- From the flip-flop input functions and the characteristic tables of the flip-flops, we obtain the next states of the flip-flops.

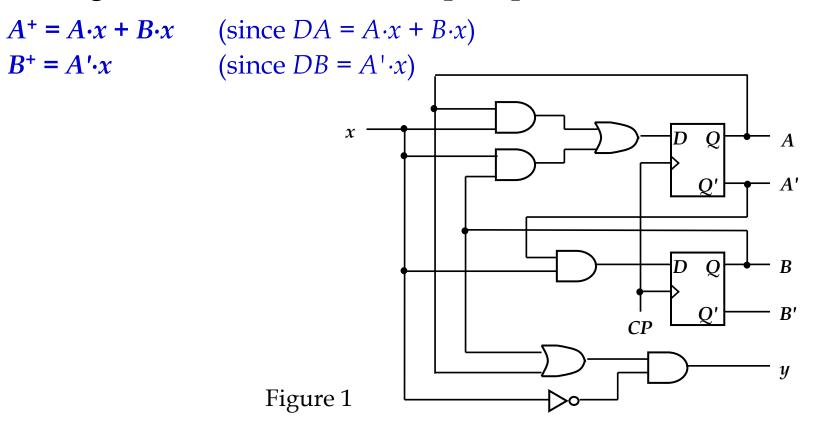
Flip-Flop Input Function (2/3)

- Example: circuit with a *JK* flip-flop.
- We use 2 letters to denote each flip-flop input: the first letter denotes the input of the flip-flop (*J* or *K* for *J*-*K* flip-flop, *S* or *R* for *S*-*R* flip-flop, *D* for *D* flip-flop, *T* for *T* flip-flop) and the second letter denotes the name of the flip-flop.



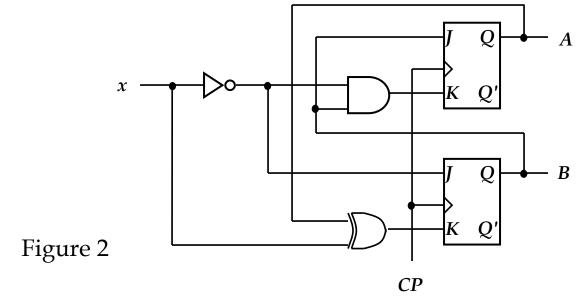
Flip-Flop Input Function (3/3)

• In Figure 1, we obtain the following state equations by observing that *Q*⁺ = *DQ* for a *D* flip-flop:



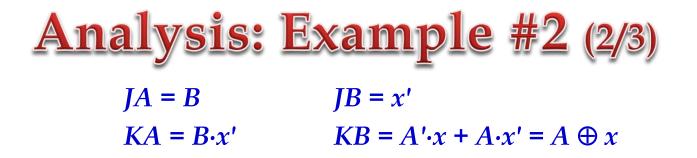
Analysis: Example #2 (1/3)

• Given Figure 2, a sequential circuit with two *J*-*K* flip-flops *A* and *B*, and one input *x*.



Obtain the flip-flop input functions from the circuit:

JA = BJB = x' $KA = B \cdot x'$ $KB = A' \cdot x + A \cdot x' = A \oplus x$



 Fill the state table using the above functions, knowing the characteristics of the flip-flops used.

J	Κ	Q(t+1)	Comments									
0	0	Q(t)	No change	Pre	sent		Ne	ext				
0	1	0	Reset	sta	ate	Input	sta	ate	Fl	ip-flo	p inpu	Its
1	0	1	Set	A	В	X	A ⁺	B ⁺	JA	KA	JB	KB
1	1	Q(t)'	Toggle	- 0	0	0	-	-		0	1	
				- 0	U	U			0	U		0
				0	0	1			0	0	0	1
				0	1	0			1	1	1	0
				0	1	1			1	0	0	1
				1	0	0			0	0	1	1
				1	0	1			0	0	0	0
				1	1	0			1	1	1	1
				1	1	1			1	0	0	0

Analysis: Example #2 (3/3)

• Draw the state diagram from the state table.

Present			Ne	ext								
sta	ate	<u>Input</u>	sta	ate	FI	Flip-flop inputs						
Α	В	X	A ⁺	B ⁺	JA	KA	JB	KB				
0	0	0			0	0	1	0				
0	0	1			0	0	0	1				
0	1	0			1	1	1	0				
0	1	1			1	0	0	1				
1	0	0			0	0	1	1				
1	0	1			0	0	0	0				
1	1	0			1	1	1	1				
1	1	1			1	0	0	0				



Analysis: Example #3 (1/3)

• Derive the state table and state diagram of this circuit.

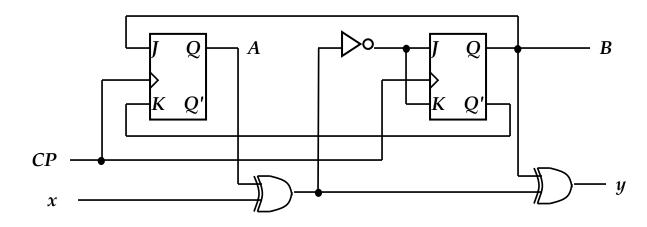


Figure 3

• Flip-flop input functions:

JA = B $JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$ KA = B'

Analysis: Example #3 (2/3)

• Flip-flop input functions:

KA = B'

 $JA = B \qquad \qquad JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$

• State table:

Pres	sent		N	ext					
<u>state</u>		<u>Input</u>	st	ate	<u>Output</u>	Fli	ip-flo	<u>ว inpเ</u>	ıts
Α	B	X	A ⁺	B ⁺	У	JA	KA	JB	KE
0	0	0			0	0	1	1	1
0	0	1			1	0	1	0	0
0	1	0			1	1	0	1	1
0	1	1			0	1	0	0	0
1	0	0			1	0	1	0	0
1	0	1			0	0	1	1	1
1	1	0			0	1	0	0	0
1	1	1		v	1	1	0	1	1

Analysis: Example #3 (3/3)

• State diagram:

Preser	nt	Next						
state	<u>Input</u>	state	<u>Output</u>	FI	ip-flo _l	<u>ρ inpι</u>	uts	
A E	3 x	$A^+ B^+$	У	JA	KA	JB	KE	
0 0	0		0	0	1	1	1	
0 0) 1		1	0	1	0	0	
0 1	0		1	1	0	1	1	
0 1	1		0	1	0	0	0	
1 0	0		1	0	1	0	0	
1 0) 1		0	0	1	1	1	
1 1	0		0	1	0	0	0	
1 1	1		1	1	0	1	1	

Flip-Flop Excitation Tables (1/2)

- *Analysis*: Starting from a circuit diagram, derive the state table or state diagram.
- *Design*: Starting from a set of specifications (in the form of state equations, state table, or state diagram), derive the logic circuit.
- *Characteristic tables* are used in analysis.
- *Excitation tables* are used in design.

Flip-Flop Excitation Tables (2/2)

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• *Excitation tables*: given the required transition from present state to next state, determine the flip-flop input(s).

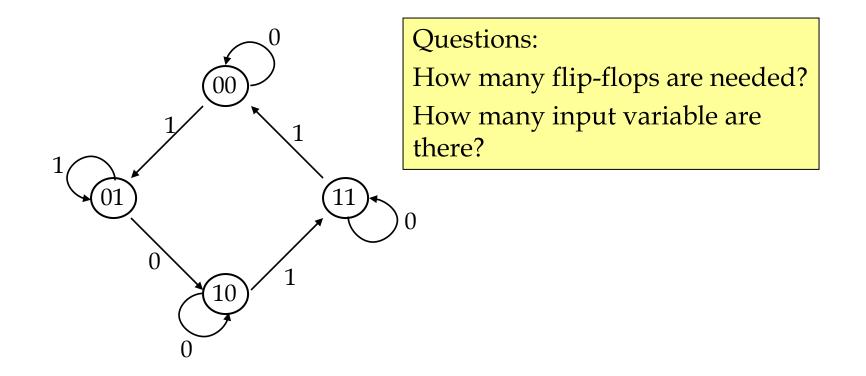
Q	Q^{+}	J	K		Q	Q	-	S	R
0	0	0	Χ	-	0	0		0	Χ
0	1	1	Χ		0	1		1	0
1	0	X	1		1	0		0	1
1	1	X	0		1	1		Χ	0
Q		p-flop	_			Q	Elip-: Q⁺		T
0		0	-			0	0		0
0	1	1				0	1		1
1	0	0				1	0		1
1	1	1				1	1		0
D			_						

Sequential Circuits: Design

- Design procedure:
 - Start with circuit specifications description of circuit behaviour, usually a state diagram or state table.
 - Derive the state table.
 - Perform state reduction if necessary.
 - Perform state assignment.
 - Determine number of flip-flops and label them.
 - Choose the type of flip-flop to be used.
 - Derive circuit excitation and output tables from the state table.
 - Derive circuit output functions and flip-flop input functions.
 - Draw the logic diagram.

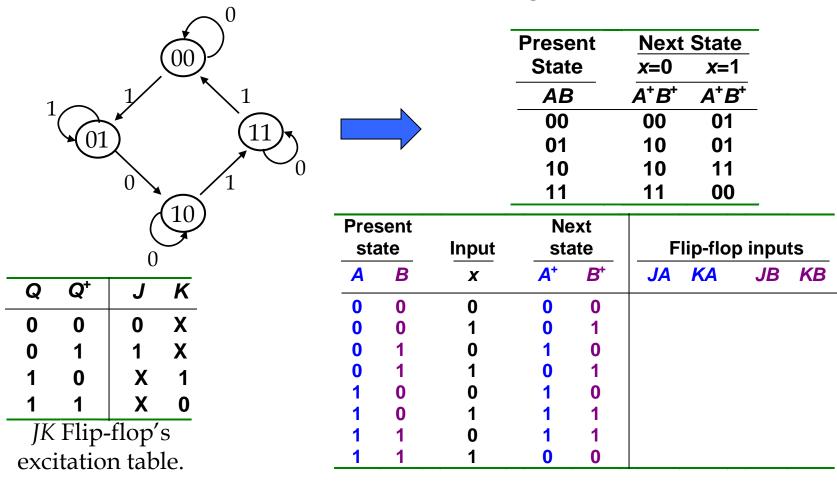
Design: Example #1 (1/5)

• Given the following state diagram, design the sequential circuit using *JK* flip-flops.



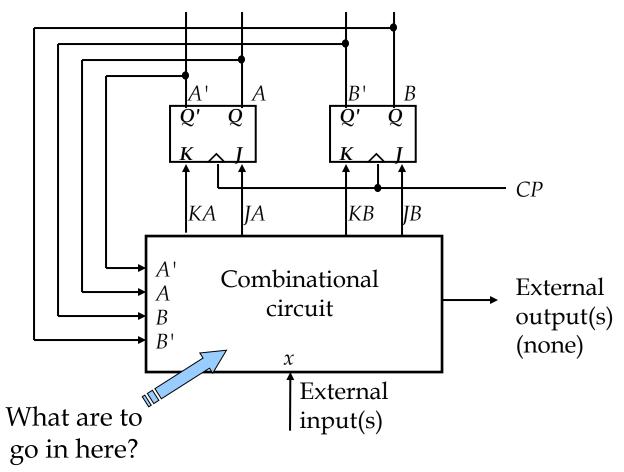
Design: Example #1 (2/5)

• Circuit state/excitation table, using *JK* flip-flops.



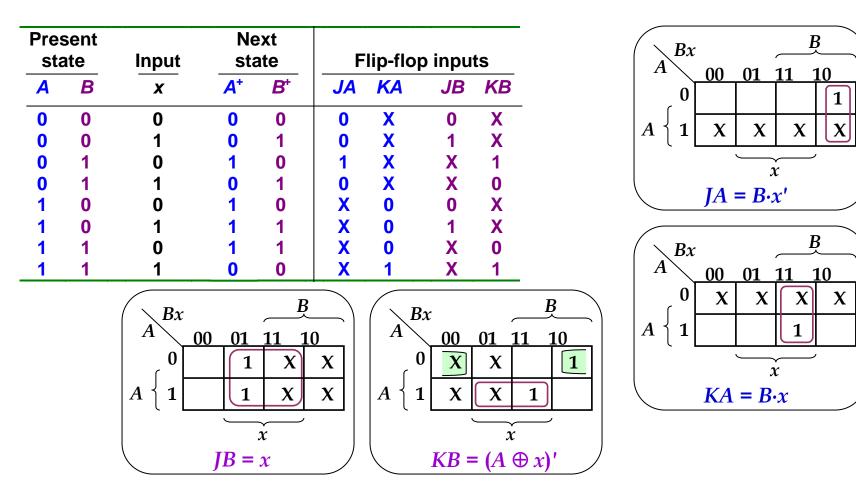
Design: Example #1 (3/5)

• Block diagram.



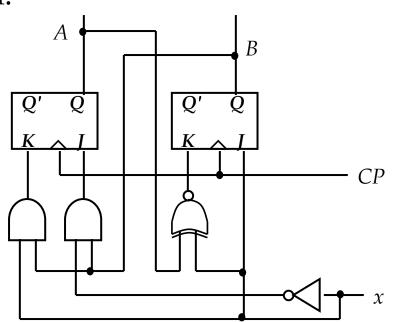
Design: Example #1 (4/5)

• From state table, get flip-flop input functions.



Design: Example #1 (5/5)

- Flip-flop input functions: $JA = B \cdot x'$ JB = x
 - $KA = B \cdot x$ $KB = (A \oplus x)'$
- Logic diagram:



Design: Example #2 (1/3)

• Using *D* flip-flops, design the circuit based on the state table below. (Exercise: Design it using *JK* flip-flops.)

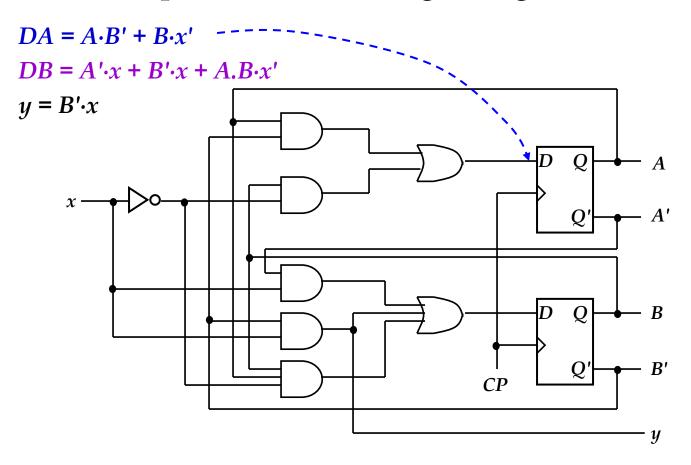
Present state A B		Input		ext ate	Output		
		X	A ⁺	B ⁺	У		
0	0	0	0	0	0		
0	0	1	0	1	1		
0	1	0	1	0	0		
0	1	1	0	1	0		
1	0	0	1	0	0		
1	0	1	1	1	1		
1	1	0	1	1	0		
1	1	1	0	0	0		

Design: Example #2 (2/3)

• Determine expressions for flip-flop inputs and the circuit



• From derived expressions, draw logic diagram:



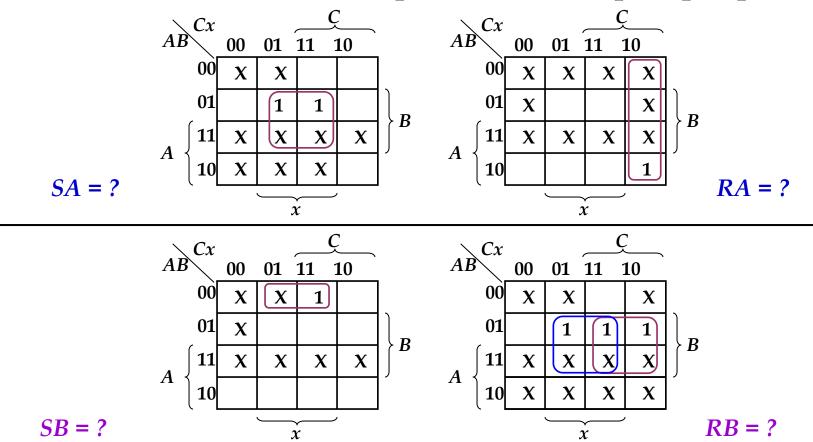
Design: Example #3 (1/4)

• Design involving unused states.

P	rese	nt			Next	:									
	state	9	Input		state			Flip-flop inputs						Output	
A	В	С	X	A ⁺	B+	C⁺	SA	RA	SB	RB	SC	RC		У	
0	0	1	0	0	0	1	0	X	0	Х	Х	0		0	
0	0	1	1	0	1	0	0	X	1	0	0	1		0	
0	1	0	0	0	1	1	0	X	Х	0	1	0		0	
0	1	0	1	1	0	0	1	0	0	1	0	Х		0	
0	1	1	0	0	0	1	0	X	0	1	Х	0		0	
0	1	1	1	1	0	0	1	0	0	1	0	1		0	
1	0	0	0	1	0	1	X	0	0	Х	1	0		0	
1	0	0	1	1	0	0	X	0	0	Х	0	Х	-	1	
1	0	1	0	0	0	1	0	1	0	Х	Х	0	-	0	
1	0	1	1	1	0	0	X	0	0	Х	0	1		1	
	Given these Derive these Are there of unused sta														
Unu	sec	l sta	ate 000:												_
0	0	0	0	X	(X	X	X	X	X	X		(X	X	
0	0	0	1	X	X	X	X	X	Х	X		(Χ	Х	_

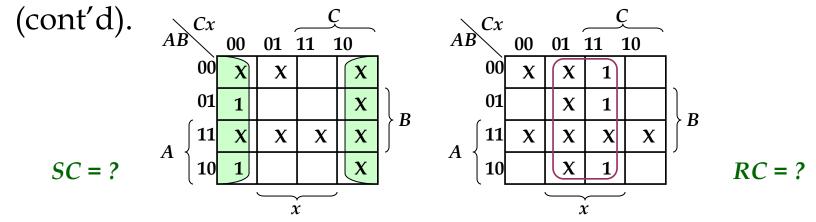
Design: Example #3 (2/4)

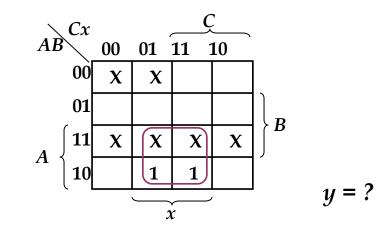
• From state table, obtain expressions for flip-flop inputs.



Design: Example #3 (3/4)

• From state table, obtain expressions for flip-flop inputs





Design: Example #3 (4/4)

• From derived expressions, draw the logic diagram:

