IKI30820 – Logic Programming
Lists, Operators, Arithmetics
Slide 04

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Outline

Representation of Lists I

- List is a simple data structure widely used in programming
- A list is a **sequence** of **any** number of items
- A list in Prolog: `[java,c,haskell,prolog]`
  - External appearance
  - It is a *structured* object, represented as a tree
- List is either empty or non-empty
- Empty list is written as Prolog **atom**: `[]`
- Non-empty list can be viewed as consisting of two things:
  - The first item, called the **head** of the list
  - The remaining part of the list, called the **tail**

Representation of Lists II

- For the list: `[java,c,haskell,prolog]`
  - Head of the list: `java`
  - Tail of the list: `[c,haskell,prolog]`
- Head can be any Prolog object, but the tail has to be a list
- This is also a list: [[a,b],c,d]
  - Head: `[a,b]`
  - Tail: `[c,d]`
- What is the head and the tail of `[[a,b,c]]`?

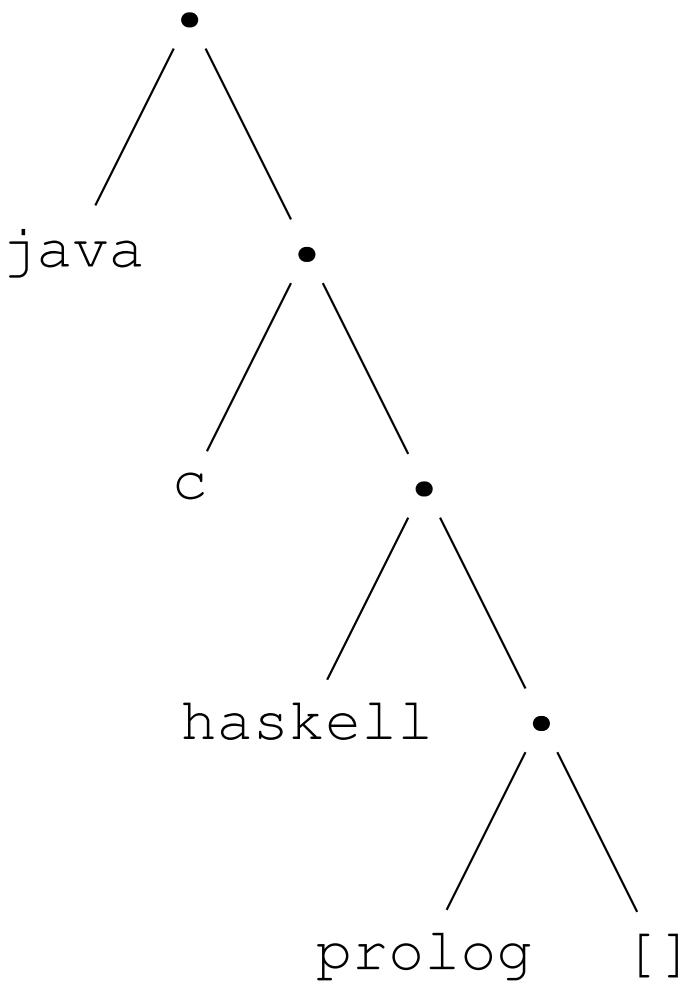
The Functor "." I

- List is a structured object
  - The head and the tail are combined into a structure by a special functor "."
  - A list is represented by: `.(Head,Tail)`
- Tail is in turn a list: either empty or it has its own head and tail
- Example: `[java,c,haskell,prolog]` is actually represented as the term:

  `.(java,.(c,.(haskell,.(prolog,[]))))`

The Functor "." II

- The corresponding tree structure for
  `.(java,.(c,.(haskell,.(prolog,[]))))`



List notation I

- The square bracket notation for list is obviously more convenient.
  - But, internally lists are represented as binary trees
- Prolog always displays a list in square bracket notation

```
?- List = .(a,.(b,.(c,[]))).

?- is_list([a.b]).
```

List notation II

- Prolog provides another notational extension, the vertical bar, which separates the head and the tail
  - We can have one or more elements followed by "`|`" and the list of remaining items
- Alternative ways of writing `[a,b,c]` are
  - `[a,|[b,c]]`
  - `[a,b|[c]]`
  - `[a,b,c|[ ]]`

```
?- [Head|Tail] = [a,b,c].
```

Built-in predicates for lists: membership I

- Membership

  `member(X,L)`

  is true when the object X is a member of the list L

```
?- member(b,[a,b,c]).

?- member(b,[a,[b,c]]).

?- member([b,c],[a,[b,c]]).
```

Built-in predicates for lists: membership II

- The program for membership relation can be based on the following observation:
  X is a member of L if either:
  1. X is the head of L, or
  2. X is the member of the tail of L
- This can be written in two clauses; the first is a simple fact and the second is a rule

```
member(X,[X|Tail]).
member(X,[Head|Tail]) :- member(X,Tail).
```

Built-in predicates for lists: list concatenation I

- `append(L1,L2,L3)` is true if the list L3 is the concatenation of the list L1 and the list L2

```
?- append([a,b],[b,c,d],[a,b,b,c,d]).

?- append([a,b],[b,c,d],[a,b,c,d]).

?- append([a,b],[1,2,3],L).
```

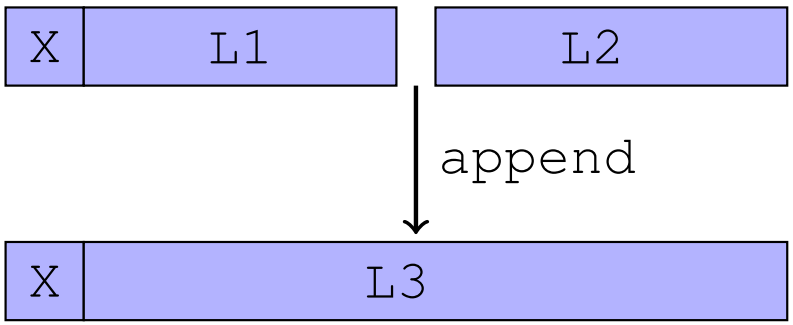
Built-in predicates for lists: list concatenation II

- The definition of append, depending on the first argument, is as follows:
  1. If the first argument is the empty list, then the second and the third must be the same list

     ```
     append([],L,L).
     ```

  2. If the first argument is a non-empty list, then it has a head and a tail of the form [X | L1]. The concatenation of [X | L1] and L2 is the list [X | L3], where L3 is the concatenation of L1 and L2. (See picture)

     ```
     append([X|L1],L2,[X|L3]) :- append(L1,L2,L3).
     ```



Give the answers of this query

```
append([],L,L).
append([X|L1],L2,[X|L3]) :- append(L1,L2,L3).

?- append(L1,L2,[a,b,c]).
```

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```
member1(X,L) :- append([X|L1],L2,L).
```

- Try with both versions of `member1`:

```
?- member1(a,[a,b,c]).

?- member1(b,[a,b,c]).
```

Example: Permutation

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2. If the first list is not empty, then it is of the form [X | L] and a permutation L2 of such list is obtained:
   - if T1 is a permutation of L; and
   - L2 is a list resulting from inserting X to any position in T1.

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   - L2 is a list resulting from inserting X to any position in T1.

How then you define `insert(X,T,L)` which is true if the list L is obtained by inserting X to a position in the list T?

```
del(X,[X|Tail],Tail).
del(X,[Y|Tail],[Y|T]) :- del(X,Tail,T).

insert(X,Y,Z) :- del(X,Z,Y).

permutation([],[]).
permutation([X|L],L2) :-
    permutation(L,T1),
    insert(X,T1,L).

?- permutation([a,b,c],[c,a,b]).

?- permutation([a,b,c],X).
```

Outline

Operators

- Operators are defined to improve the readability of source-code
- For example, without operators, to write `a*b+c*d`, one would have to write: `+(*(a,b),*(c,d))`
- A number of operators have been predefined. All operators, except for the comma (,) can be redefined by the user.

1200	xfx	-->, :-
1200	fx	:-, ?-
1150	fx	dynamic, discontiguous, initialization, module_transparent, multifile, thread_local, volatile
1100	xfy	;, |
1050	xfy	->, **op***->
1000	xfy	,
954	xfy	\
900	fy	\+
900	fx	~
700	xfx	<, =, =.., =@=, =:=, =<, ==, =\=, >, >=, @<, @=<, @>, @>=, \=, \==, **is**
600	xfy	:
500	yfx	+, -, /\, \/, xor
500	fx	+, -, ?, \
400	yfx	*, /, //, rdiv, <<, >>, **mod**, rem
200	xfx	**
200	xfy	^

Defining new operators I

op(+Precedence, +Type, **:Name**)

Declares **Name** to be an operator of type `Type` with precedence `Precedence`

- **Name** can also be a list of names, in which case all elements of the list are declared to be identical operators
- `Precedence` is an integer between 0 and 1200. Precedence 0 removes the declaration. The higher precedence the lower the binding power of the operator.
- `Type` is one of: $x\,f$, $y\,f$, $x\,f\;x$, $x\,f\;y$, $y\,f\;x$, $f\;y$, or $f\;x$
- The 'f' indicates the position of the functor, while x and y indicate the position of the arguments (postfix: $x\,f$, $y\,f$, infix: $x\,f\;x$, $y\,f\;x$, $x\,f\;y$, prefix: $f\;x$, $f\;y$)

Defining new operators II

- 'y' should be interpreted as "on this position a term with precedence lower or equal to the precedence of the functor should occur"
- For 'x' the precedence of the argument must be strictly lower than the precedence of the functor
- The precedence of a term is 0, unless its principal functor is an operator, in which case the precedence is the precedence of this operator
- A term enclosed in brackets (...) has precedence 0

Example

Using predefined operator '-', how would 5-2-1 be interpreted?

- Operator '-' is defined with precedence as 500 and type yfx.
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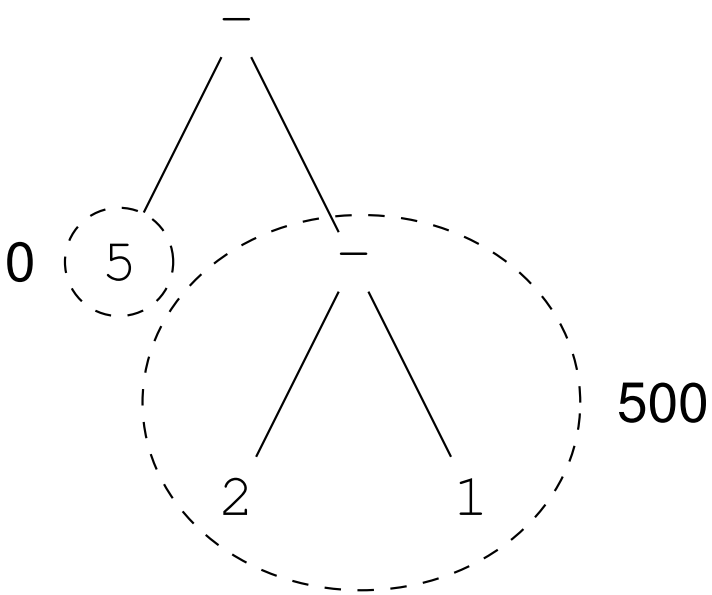

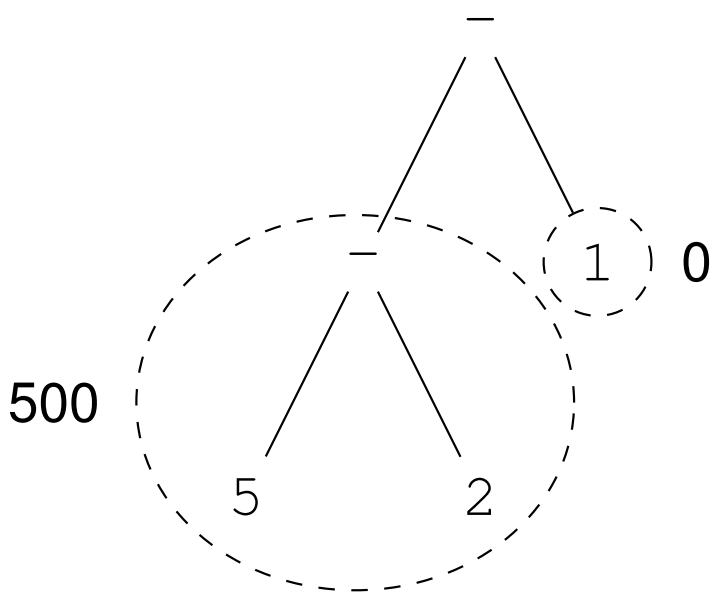

- Programmer can define new operators byinserting into the program special kinds of clauses, called **directives**, starting with "`:-`"
- Note that operator definition do not specify any operation or action

```
:- op(650,xfx,suka).
unyil suka bakso.
iin suka tempe.
unyil suka mie.

budi suka X :- unyil suka X.
budi suka X :- X suka tempe.
```

Exercise

- Write an operator definition for "`dan`" so that we can write:

```
unyil suka bakso dan mie dan coklat
```

- Add a rule to the following program "suka":

```
unyil suka bakso.
iin suka tempe.
unyil suka mie.
unyil suka coklat.
```

so that we can ask queries like:

```
?- Siapa suka bakso dan mie.
Siapa = unyil
Yes

?- Siapa suka bakso dan coklat dan mie.
Siapa = unyil
Yes
```

`current_op(?Precedence, ?Type, ?:`**`Name`**`)`

succeeds when **`Name`** is currently defined as an operator of type `Type` with precedence `Precedence`.

```
?- current_op(P,T,suka).
P = 650
T = xfx
Yes

?- current_op(P,T,-).
P = 500
T = yfx
Yes
```

More example

- De Morgan's equivalence theorem:

$$\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$$

- Can be expressed in Prolog term as:

```
equivalence(not(and(A,B)),or(not(A),not(B)))
```

- With a suitable set of operators

```
:- op(800, xfx, <===> ).
:- op(700, xfy, v).
:- op(600, xfy, &).
:- op(500, fy, ~).
```

- The formula can now be written as:

```
~(A & B) <===> ~A v ~B.
```

Outline

Arithmetic

- The general arithmetic predicates all handle expressions.
- An expression is either a simple number or a function. The arguments of a function are expressions.
- Note that '=' does not cause any arithmetic operation. Use '**is**' to activate arithmetic operation.

```
?- Y = 1+3.

?- Y is 1+3.
```

Predicate: `is`

`Number` **`is`** `+Expr`

- Succeeds when `Number` has successfully been unified with the number which `Expr` evaluates to.
- Note that normally, **`is`**/2 will be used with unbounded left operand. If equality is to be tested,=:=/2 should be used.

Arithmetic predicates

- `between(+Low, +High, ?Value)`
  `Low` and `High` are integers. `High` $\geq$ `Low`. If `Value` is an integer, `Low` $\leq$ `Value` $\leq$ `High`. When `Value` is a variable, it is successively bound to all integers between `Low` and `High`.
- `succ(?Int1, ?Int2)`
  succeeds if `Int2 = Int1 + 1`. At least one of the arguments must be instantiated to an integer
- `plus(?Int1, ?Int2, ?Int3)`
  succeeds if `Int3=Int1+Int2`. At least two of the three arguments must be instantiated to integers.

Other arithmetic predicates

- `Expr1 > Expr2`
  succeeds when expression `Expr1` evaluates to a larger number than `Expr2`
- `Expr1 < Expr2`
- `Expr1 =< Expr2`
- `Expr1 >= Expr2`
- `Expr1 =\= Expr2`
  succeeds when expression `Expr1` evaluates to a number non-equal to `Expr2`
- `Expr1 =:= Expr2`
  succeeds when expression `Expr1` evaluates to a number equal to `Expr2`

Arithmetic function

Arithmetic functions are terms which are evaluated by the arithmetic predicates, such as '**`is`**'

- `-Expr` minus
- `Expr1 + Expr2` addition
- `Expr1 - Expr2` subtraction
- `Expr1 * Expr2` multiplication
- `Expr1 / Expr2` division
- `Expr1 // Expr2` Integer division

Other functions I

- Power: `Expr1 ** Expr2` (the same as ^)
- `IntExpr1` **`mod`** `IntExpr2`
  Result: `IntExpr1-(IntExpr1//IntExpr2)*IntExpr2`
- Remainder of division: `IntExpr1 rem IntExpr2`
  Result: `float_fractional_part(IntExpr1/IntExpr2)`
- `exp(Expr)`. Result: e^{Expr}
- `abs(Expr)`, `sign(Expr)`, `max(Expr1,Expr2)`, `min(Expr1,Expr2)`
- `random(Int)`
  evaluates to random integer i for which $0 \leq i < Int$. The seed of this random generator is determined by the system clock when SWI- Prolog started
- `round(Expr)`, `float(Expr)`, `truncate(Expr)`, `floor(Expr)`, `ceiling(Expr)`, `sqrt(Expr)`

Other functions II

- `sin(Expr), cos(Expr), tan(Expr)`
  `Expr` is in radian
- `asin(Expr), acos(Expr), atan(Expr)`
  Result is the angle in radian
- `log(Expr)`
  Result: $\ln Expr$
- `log10(Expr)`
  Result: $\log Expr$
- `pi`
  evaluates to the mathematical constant π (3.14159)
- `e`
  evaluates to the mathematical constant e (2.71828)

Other functions III

- `IntExpr1 >> IntExpr2`
  Bitwise shift `IntExpr1` by `IntExpr2` bits to the right
- `IntExpr1 << IntExpr2`
  Bitwise shift `IntExpr1` by `IntExpr2` bits to the left
- `IntExpr1 \/ IntExpr2`
  Bitwise 'or' `IntExpr1` and `IntExpr2`
- `IntExpr1 /\ IntExpr2`
  Bitwise 'and' `IntExpr1` and `IntExpr2`
- `IntExpr1 xor IntExpr2`
  Bitwise 'exclusive or' `IntExpr1` and `IntExpr2`
- `\ IntExpr`
  Bitwise negation

Example

```
?- 1+3 =:= 3+1.

?- 1+3 == 3+1.

?- 1+A=B+3.

?- X is sin(pi/2).
```

Example: Finding GCD

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1. If X and Y are equal, then D is equal to X
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```
gcd(X,X,X).
gcd(X,Y,D) :- X<Y, Y1 is Y-X, gcd(X,Y1,D).
gcd(X,Y,D) :- Y<X, gcd(Y,X,D).
```

Example: length of a list

- **`length`**`(List,N)`
  succeeds when N is the number of elements in the list List

Example: length of a list

- `length(List,N)`
  succeeds when N is the number of elements in the list List
- Two cases:
  1. If the list is empty then its length is 0
  2. If the list is not empty then `List = [Head|Tail]`; and its length is equal to 1 plus the length of the tail `Tail`

```
length([],0).
length([_|Tail],N) :- length(Tail,N1), N is N+1.
```